Complex Numbers

Question1

ω is a complex cube root of unity and Z is a complex number satisfying $|Z-1| \leq 2$. The possible values of r such that $|Z-1| \leq 2$ and $\left|\omega Z-1-\omega^2\right|=r$ have no common solution are

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Options:

A.

$0 \leq r \leq 4$

B.

$r=|\omega|$ only

C.

$r>4$

D.

$1

Answer: C

Solution:

We have, $|z-1| \leq 2$

$$
\begin{aligned}
&And \left|\omega z-1-\omega^2\right|=r \\
&\quad |\omega|\left|z-\omega^2-\omega\right|=r \\
&\quad |z+1|=r \\
&\qquad \left[\because |\omega|=1, 1+\omega+\omega^2=0\right] \\
&\quad |z-1| \leq 2 \Rightarrow |z+1-2| \leq 2 \\
&\quad |z+1|-2 \leq 2 \Rightarrow |z+1| \leq 4
\end{aligned}
$$

For no solution

$\therefore \quad |z+1|>4 \Rightarrow r>4$

Question2

If $|Z|=2, Z_1=\frac{Z}{2} e^{i \alpha}$ and θ is the $\operatorname{amp}(Z)$, then $\frac{Z_1^n-Z_1^{-n}}{Z_1^n+Z_1^{-n}}=$

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Options:

A.

$2^n i \tan (n \theta+n \alpha)$

B.

$i \tan (n \theta-n \alpha)$

C.

$i \tan (n \theta+n \alpha)$

D.

$\tan (n \theta+n \alpha)$

Answer: C

Solution:

Given, $|z|=2, z_1=\frac{z}{2} e^{i \alpha}$

$\therefore \quad z=2 e^{i \theta}$

(θ is the argument of z)

And $z_1 = \frac{2e^{i\theta}}{2} e^{i\alpha} = e^{i(\theta+\alpha)}$

Now, $z_1^n = e^{in(\theta+\alpha)}$

$\Rightarrow \quad z_1^{-n} = e^{-in(\theta+\alpha)}$

We know that $\frac{e^{ix}-e^{-ix}}{e^{ix}+e^{-ix}} = i \tan x$

$$\begin{aligned} \therefore \frac{z_1^n - z_1^{-n}}{z_1^n + z_1^{-n}} &= \frac{e^{in(\theta+\alpha)} - e^{-in(\theta+\alpha)}}{e^{in(\theta+\alpha)} + e^{-in(\theta+\alpha)}} \\ &= i \tan(n(\theta+\alpha)) \\ &= i \tan(n\theta + n\alpha) \end{aligned}$$

Question3

If $n, K \in N$ such that $n \neq 3K$, then $(\sqrt{3}+i)^{2n} + (\sqrt{3}-i)^{2n} =$

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Options:

A.

$(-1)^n 2^{2n+1}$

B.

$(-1)^{n+1} 2^{2n+1}$

C.

$(-1)^{n+1} 2^{2n}$

D.

$(-1)^{n+1} 2^n$

Answer: C

Solution:

Given, $n, K \in N$ such that $n \neq 3K$

Let $z = \sqrt{3} + i$, then $\bar{z} = \sqrt{3} - i$

$|z| = |\bar{z}| = \sqrt{3+1} = 2$

$\arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\arg(z) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

$\therefore \quad z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\Rightarrow z^{2n} = 2^{2n}\left(\cos\frac{2n\pi}{6} + i\sin\frac{2n\pi}{6}\right)$

And $\bar{z} = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$\Rightarrow \bar{z}^{2n} = 2^{2n}\left(\cos\frac{2n\pi}{6} - i\sin\frac{2n\pi}{6}\right)$

$\therefore z^{2n} + \bar{z}^{2n} = 2 \cdot 2^{2n}\cos\left(\frac{n\pi}{3}\right)$

$= 2^{2n+1}\cos\left(\frac{n\pi}{3}\right)$

Since, $n \neq 3K \Rightarrow n$ is not a multiple of 3 $\cos\left(\frac{n\pi}{3}\right) \neq \cos(k\pi) = \pm 1 = (-1)^{n+1}2^{2n}$

Question4

In argand plane, no value of $\sqrt[3]{1 - i\sqrt{3}}$ lie in

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Options:

A.

First quadrant

B.

second quadrant

C.

Third quadrant

D.

Fourth quadrant

Answer: A

Solution:

Let $z = 1 - i\sqrt{3}$

$$\therefore \quad |z| = \sqrt{1 + (\sqrt{3})^2} = 2$$

$$\arg(z) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\tan\frac{\pi}{3}$$

$$\therefore \quad z = 2\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$= 2e^{-i\pi/3}$$

$$\therefore \quad \sqrt[3]{z} = 2^{1/3} \cdot e^{4 - i\pi/3 + 2k\pi i}$$

$$= 2^{1/3} e^{i\left(-\frac{\pi}{9} + \frac{2k\pi}{3}\right)}$$

For any integer $k \in \{0, 1, 2\}$

$\omega_0 =$ lies in 4th quadrant

$\omega_1 =$ lies in 2nd quadrant

$\omega_2 =$ lies in 3rd quadrant

Question5

If $\frac{2+3i}{i-2} - \frac{4i-3}{3+4i} = x + iy$, then $3x + y =$

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Options:

A.

4

B.

-4

C.

-2

D.

2

Answer: B

Solution:

Given, $\frac{2+3i}{i-2} - \frac{4i-3}{3+4i} = x + iy$

Now, $\frac{2+3i}{i-2} = \frac{2+3i}{-2+i} \times \frac{(-2-i)}{(-2-i)}$

$\Rightarrow \quad \frac{-4-2i-6i-3i^2}{(-2)^2-i^2} = \frac{-1-8i}{5}$

And, $\frac{4i-3}{3+4i} \times \frac{3-4i}{3-4i}$

$\Rightarrow \quad \frac{-9+12i+12i-16i^2}{3^2-(4i)^2} = \frac{7+24i}{25}$

So, $\quad \frac{2+3i}{i-2} - \frac{4i-3}{3+4i}$

$$= \frac{-1}{5} - \frac{8i}{5} - \frac{7}{25} - \frac{24}{25}i$$
$$= \frac{-12}{25} - \frac{64}{25}i$$

On comparing, we get

$x = \frac{-12}{25}, y = \frac{-64}{25}$

So, $3x + y = 3\left(\frac{-12}{25}\right) - \frac{64}{25}$

$= -\frac{36}{25} - \frac{64}{25} = \frac{-100}{25} = -4$

Question6

Let $z = x + iy$ and $P(x, y)$ be a point on the argand plane. If z satisfies the condition $\arg\left(\frac{z-3i}{z+2i}\right) = \frac{\pi}{4}$, then the locus of P is

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Options:

A.

$x^2 + y^2 - y - 6 = 0, (x, y) \neq (0, -2)$

B.

$x^2 + y^2 - x - y - 6 = 0, (x, y) \neq (0, -2)$

C.

$x^2 + y^2 + 5x - y - 6 = 0, (x, y) \neq (0, -2)$

D.

$x^2 + y^2 + x - y - 6 = 0, (x, y) \neq (0, -2)$

Answer: C

Solution:

Given, arg $\left(\frac{z-3i}{z+2i}\right) = \frac{\pi}{4}$

Let $z = x + iy$

So, $\frac{z-3i}{z+2i} = \frac{x+iy-3i}{x+iy+2i}$

$\Rightarrow \frac{x+i(y-3)}{x+i(y+2)} \times \frac{x-i(y+2)}{x-i(y+2)}$

$$\Rightarrow \frac{x^2 - ix(y+2) + ix(y-3) + (y-3)(y+2)}{x^2 + (y+2)^2}$$
$$\Rightarrow \frac{x^2 + y^2 - y - 6 - i5x}{x^2 + (y+2)^2}$$

Let the complex numbers be $A + iB$, where

$A = \frac{x^2+y^2-y-6}{x^2+(y+2)^2}$ and $B = \frac{-5x}{x^2+(y+2)^2}$

So, arg $\left(\frac{z-3i}{z+2i}\right) = \left(\frac{\pi}{4}\right)$

$$\Rightarrow \quad \tan\left(\frac{B}{A}\right) = \frac{\pi}{4}$$
$$\Rightarrow \quad \frac{-5x}{x^2 + y^2 - y - 6} = \tan^{-1}\left(\frac{\pi}{4}\right) = 1$$
$$\Rightarrow \quad x^2 + y^2 - y - 6 = -5x$$
$$\Rightarrow \quad x^2 + 5x + y^2 - y - 6 = 0, \text{ where}$$
$x \neq 0$ and $\quad y \neq -2$

Question7

If ω is a complex cube root of unity and $x = \omega^2 - \omega + 2$, then

Options:

A.

$x^2 - 4x + 7 = 0$

B.

$x^2 + 4x + 7 = 0$

C.

$x^2 - 2x + 4 = 0$

D.

$x^2 + 2x + 4 = 0$

Answer: A

Solution:

Given, $x = \omega^2 - \omega + 2$ where w is a complex cube root of unity.

We know that $1 + \omega + \omega^2 = 0$

$\Rightarrow \omega^2 = -1 - \omega$

So, $x = (-1 - \omega) - \omega + 2 = 1 - 2\omega$

$\Rightarrow \omega = \frac{1-x}{2}$

So, $1 + \omega + \omega^2 = 0$

$$\begin{aligned}
\Rightarrow \quad & 1 + \left(\frac{1-x}{2}\right) + \left(\frac{1-x}{2}\right)^2 = 0 \\
\Rightarrow \quad & 1 + \frac{(1-x)}{2} + \frac{(1-x)^2}{4} = 0 \\
\Rightarrow \quad & 4 + 2(1-x) + 1 - 2x + x^2 = 0 \\
\Rightarrow \quad & 4 + 2 - 2x + 1 - 2x + x^2 = 0 \\
\Rightarrow \quad & x^2 - 4x + 7 = 0
\end{aligned}$$

Question8

The product of all the values of $(\sqrt{3}-i)^{\frac{3}{7}}$ is

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Options:

A.

8

B.

-8

C.

$8i$

D.

$-8i$

Answer: D

Solution:

Let $z=\sqrt{3}-i$

$|z|=\sqrt{(\sqrt{3})^2+(-1)^2}=\sqrt{3+1}=2$

And $\quad \theta=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=-\frac{\pi}{6}$ or $\frac{11\pi}{6}$

So, $\quad z=2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$

Now, $z^{\frac{3}{7}}=(\sqrt{3}-i)^{\frac{3}{7}}$

First, $z^3=(\sqrt{3}-i)^3$

$$=2^3\left(\cos\left(\frac{-3\pi}{6}\right)+i\sin\left(\frac{-3\pi}{6}\right)\right)$$
$$=8\left(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}\right) \qquad \ldots(i)$$

Now, $z^{\frac{3}{7}}=(\sqrt{3}-i)^{\frac{3}{7}}$

$$=(8)^{\frac{1}{7}}\left(\cos\left(\frac{-\frac{\pi}{2}+2k\pi}{7}\right)+i\sin\left(\frac{-\frac{\pi}{2}+2k\pi}{7}\right)\right) \qquad \ldots(ii)$$

for $k=0,1,\ldots,6$

So, $z^3=8(0-i)=-8i$ [Using Eq. (i)]

Now, we know that the product of nt h root of a complex number A is given by $(-1)^{n-1}A$, if A is a real number.

Since, $n=7$ is odd, the product of the 7 roots of A is A itself.

$\therefore$ The product of the 7th roots of A is A itself.

$\therefore \quad (\sqrt{3}-i)^{\frac{3}{7}}=z^3$

So, product $=-8i$

Question9

If $z=\frac{(2-i)(1+i)^3}{(1-i)^2}$, then $\arg(z)=$

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Options:

A. $\tan^{-1}\left(\frac{1}{3}\right)-\pi$

B. $\tan^{-1}\left(\frac{3}{4}\right)-\pi$

C. $\pi-\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{1}{3}\right)$

Answer: A

Solution:

$z=\frac{(2-i)(1+i)^3}{(1-i)^2}$

$$z=\frac{(2-i)\left(1+i^3+3i^2+3i\right)}{1+i^2-2i}$$

$$=\frac{(2-i)(1-i-3+3i)}{-2i}$$

$$=\frac{(2-i)(-2+2i)}{-2i}=\frac{(2-i)(1-i)}{i}$$

$$= \frac{2 - i - 2i + i^2}{i}$$

$$= \frac{1 - 3i}{i} = -i - 3$$

$$z = -3 - i$$

$$\arg z = \tan^{-1}\left(\frac{1}{3}\right) - \pi$$

Question10

$z = x + iy$ and the point P represents z in the argand plane. If the amplitude of $\left(\frac{2z-i}{z+2i}\right)$ is $\frac{\pi}{4}$, then the equation of the locus of P is

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Options:

A. $2x^2 + 2y^2 - 3x + 3y - 2 = 0, (x, y) \neq (0, -2)$

B. $2x^2 + 2y^2 + 5x + 3y - 2 = 0, (x, y) \neq 0, -2)$

C. $2x^2 + 2y^2 + 3x + 3y - 2 = 0, (x, y) \neq 0, 2)$

D. $2x^2 + 2y^2 - 5x + 3y - 2 = 0, (x, y) \neq (0, 2)$

Answer: B

Solution:

$$\operatorname{amp}\left(\frac{2z-i}{z+2i}\right) = \frac{\pi}{4} \quad \text{(given)}$$

$$\Rightarrow \operatorname{amp}(2z - i) - \operatorname{amp}(z + 2i) = \frac{\pi}{4}$$

$$\Rightarrow \operatorname{amp}[2x + i(2y - 1)] - \operatorname{amp}[x + i(y + 2)] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{2y-1}{2x}\right) - \tan^{-1}\left(\frac{y+2}{x}\right) = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1}\left(\frac{2y-1}{2x}\right) = \tan^{-1} 1 + \tan^{-1}\left(\frac{y+2}{x}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2y-1}{2x}\right) = \tan^{-1} \frac{1+\frac{y+2}{x}}{1-\frac{y+2}{x}}$$

$$\Rightarrow \quad \frac{2y-1}{2x} = \frac{x+y+2}{x-y-2}$$

$$\Rightarrow (2y-1)(x-y-2) = 2x(x+y+2)$$

$$\Rightarrow 2xy - 2y^2 - 4y - x + y + 2 = 2x^2 + 2xy + 4x$$

$$\Rightarrow 2x^2 + 2y^2 + 5x + 3y - 2 = 0, (x, y) \neq (0, -2).$$

Question11

α, β are the roots of the equation $x^2 + 2x + 4 = 0$. If the point representing α in the argand diagram lies in the 2nd quadrant and $\alpha^{2024} - \beta^{2024} = ik, (i = \sqrt{-1})$, then $k =$

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Options:

A. $-2^{2025}\sqrt{3}$

B. $2^{2025}\sqrt{3}$

C. $-2^{2024}\sqrt{3}$

D. $2^{2004}\sqrt{3}$

Answer: C

Solution:

$x^2 + 2x + 4 = 0$

$$x = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2}$$

$\alpha = 2e^{i\frac{2\pi}{3}}$

$x = -1 \pm i\sqrt{3} = 2\left(\frac{\cos 2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

and $\quad \beta = 2e^{-i\frac{2\pi}{3}}$

$\alpha^{2024} - \beta^{2024} = \left[\frac{\alpha^{2025}}{\alpha} - \frac{\beta^{2025}}{\beta}\right]$

$= \frac{2^{2025}e^{i\frac{2\pi}{3}\times 2025}}{\alpha} - \frac{2^{2025}e^{-i\frac{2\pi}{3}\times 2025}}{\beta}$

$= 2^{2025}\left[\frac{e^{i1350\pi}}{\alpha} - \frac{e^{-i1350\pi}}{\beta}\right]$

$= 2^{2025}\left[\frac{1}{\alpha} - \frac{1}{\beta}\right] = 2^{2025}\left(\frac{1}{2}e^{-i\frac{2\pi}{3}} - \frac{1}{2}e^{-i\frac{i\pi}{3}}\right)$

$= -2^{2024}\left(e^{i\frac{2\pi}{3}} - e^{-i\frac{2\pi}{3}}\right)$

$= -2^{2024}\left(i2\sin\frac{2\pi}{3}\right)$

$= -2^{2024}\cdot 2 \times \frac{\sqrt{3}}{2}i = iK$

$\therefore \quad K = -2^{2024}\sqrt{3}$

Question12

If $z = x + iy$ satisfies the equation $z^2 + az + a^2 = 0, a \in R$, then

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Options:

A. $|z| = |a|$

B. $|z - a| = |a|$

C. $z = |a|$

D. $z = a$

Answer: A

Solution:

To solve the given equation $z^2 + az + a^2 = 0$, where $z = x + iy$ and $a \in \mathbb{R}$, follow these steps:

First, multiply both sides of the equation by $(z - a)$:

$(z - a)(z^2 + az + a^2) = 0$

Apply the identity for the difference of cubes:

$z^3 - a^3 = 0$

This implies:

$z^3 = a^3$

Taking the modulus (or absolute value) of both sides:

$|z|^3 = |a|^3$

Thus, we conclude:

$|z| = |a|$

Question13

If z_1, z_2, z_3 are three complex numbers with unit modulus such that $|z_1 - z_2|^2 + |z_1 - z_3|^2 = 4$, then $z_1\bar{z}_2 + \bar{z}_1 z_2 + z_1\bar{z}_3 + \bar{z}_1 z_3 =$

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Options:

A. 0

B. $|z_2|^2 + |z_3|^2$

C. $|z_1|^2 - |z_2 + z_3|^2$

D. 1

Answer: A

Solution:

Given the complex numbers z_1, z_2, z_3 with unit modulus, we know:

$z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$

This implies that each complex number has a magnitude of 1. We are also given the condition:

$|z_1 - z_2|^2 + |z_1 - z_3|^2 = 4$

We can expand the squared magnitudes as follows:

$$|z_1 - z_2|^2 = (z_1 - z_2)(\bar{z}_1 - \bar{z}_2) = z_1\bar{z}_1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + z_2\bar{z}_2,$$
$$|z_1 - z_3|^2 = (z_1 - z_3)(\bar{z}_1 - \bar{z}_3) = z_1\bar{z}_1 - z_1\bar{z}_3 - \bar{z}_1 z_3 + z_3\bar{z}_3.$$

Substitute the expansions back into the equation:

$$(z_1\bar{z}_1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + z_2\bar{z}_2)$$
$$+ (z_1\bar{z}_1 - z_1\bar{z}_3 - \bar{z}_1 z_3 + z_3\bar{z}_3) = 4.$$

Since $z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$, we simplify:

$(1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + 1) + (1 - z_1\bar{z}_3 - \bar{z}_1 z_3 + 1) = 4$

Simplifying further gives:

$4 - (z_1\bar{z}_2 + \bar{z}_1 z_2 + z_1\bar{z}_3 + \bar{z}_1 z_3) = 4$

Thus:

$z_1\bar{z}_2 + \bar{z}_1 z_2 + z_1\bar{z}_3 + \bar{z}_1 z_3 = 0$

This is the required result.

Question14

If ω is the complex cube root of unity and

$\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)^k + \left(\frac{a+b\omega+c\omega^2}{b+a\omega^2+c\omega}\right)^l = 2$, then $2k + l$ is always

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Options:

A. divisible by 2

B. divisible by 6

C. divisible by 3

D. divisible by 5

Answer: C

Solution:

Given the equation:

$$\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)^k + \left(\frac{a+b\omega+c\omega^2}{b+a\omega^2+c\omega}\right)^l = 2$$

where ω is the complex cube root of unity, meaning $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.

To solve this, we first manipulate the terms by using properties of complex numbers. Consider multiplying and dividing:

Multiply and divide the first term by ω^2:

$$\left(\frac{\omega^2(a+b\omega+c\omega^2)}{\omega^2(c+a\omega+b\omega^2)}\right)^k$$

Multiply and divide the second term by ω:

$$\left(\frac{\omega(a+b\omega+c\omega^2)}{\omega(b+a\omega^2+c\omega)}\right)^l$$

This simplifies to:

$$\left(\omega^{2k}\right) + \left(\omega^l\right) = 2$$

For these powers to sum to 2, and knowing the properties of cube roots of unity, we have specific implications for the exponents:

Since ω^n can only take values 1, ω, or ω^2, for the sum $\omega^{2k} + \omega^l = 2$ to be a real number (i.e., 2), both ω^{2k} and ω^l must equal 1.

Thus, this requires both 2k and l to be multiples of 3 (since $\omega^3 = 1$).

Hence, the expression $2k + l$ is also a multiple of 3, indicating that $2k + l$ is always divisible by 3.

Question15

If $z_1 = \sqrt{3} + i\sqrt{3}$ and $z_2 = \sqrt{3} + i$, and $\left(\frac{z_1}{z_2}\right)^{50} = x + iy$, then the point (x, y) lies in

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Options:

A. first quadrant

B. second quadrant

C. third quadrant

D. fourth quadrant

Answer: A

Solution:

Given, $z_1=\sqrt{3}+i\sqrt{3}$

$$
\begin{aligned}
& z_2=\sqrt{3}+i \\
& \frac{z_1}{z_2}=\frac{\sqrt{3}(1+i)}{\sqrt{3}+i}=\frac{\sqrt{3}(1+i)}{\sqrt{3}+i} \frac{(\sqrt{3}-i)}{(\sqrt{3}-i)} \\
& \quad=\frac{\sqrt{3}\left(\sqrt{3}-i+\sqrt{3} i-i^2\right)}{3-i^2} \\
& \quad=\frac{\sqrt{3}}{4}[\sqrt{3}+1+i(\sqrt{3}-1)]
\end{aligned}
$$

multiply and divide by $2\sqrt{2}$.

$$
\begin{aligned}
\Rightarrow \quad & \frac{z_1}{z_2}=\frac{2\sqrt{2}\sqrt{3}}{4} \quad\left[\cos 15^{\circ}+i \sin 15^{\circ}\right] \\
& \frac{z_1}{z_2}=\frac{\sqrt{6}}{2}\left[e^{i 15^{\circ}}\right] \\
\Rightarrow\left(\frac{z_1}{z_2}\right)^{50} & =\left(\frac{\sqrt{6}}{2}\right)^{50} e^{i 750^{\circ}} \\
& \theta=750^{\circ}-720^{\circ}=30^{\circ}
\end{aligned}
$$

$\because$ First quadrant

Question16

The roots of the equation $x^3-3x^2+3x+7=0$ are α, β, λ and ω, ω^2 are complex cube roots of unity, If the terms containing x^2 and x are missing in the transformed equation when each one of these roots is decreased by h, then $\frac{\alpha-h}{\beta-h}+\frac{\beta-h}{\gamma-h}+\frac{\gamma-h}{\alpha-h}=$

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Options:

A. $\frac{3}{\omega^2}$

B. 3ω

C. 0

D. $3\omega^2$

Answer: D

Solution:

$x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow \quad (x+1)\left(x^2 - 4x + 7\right) = 0$

$\Rightarrow \quad x = -1$ or $\frac{4 \pm \sqrt{16-28}}{2(1)}$

$\Rightarrow \quad x = -1$ or $2 \pm \sqrt{3}i$

$\therefore \quad \alpha = -1, \beta = 2 + \sqrt{3}i$ and

$\gamma = 2 - \sqrt{3}i$

New root are $-1 - h, (2-h) + \sqrt{3}i$ and $(2-h) - \sqrt{3}i$

Equation which have these roots is $(x+1+h)(x+h-2-\sqrt{3}i)$

$(x+h-2+\sqrt{3}i) = 0$

$\Rightarrow (x+1+h)\left[(x+h-2)^2 - (\sqrt{3}i)^2\right] = 0$

$\Rightarrow (x+1+h)\left[x^2 + (h-2)^2\right.$

$\left.+2x(h-2) + 3\right] = 0$

In this equation terms containing x^2 and x should be missing.

$\therefore (1+h) + 2(h-2) = 0$ and

$2(1+h)(h-2) + 3 + (h-2)^2 = 0$

Now, $\frac{\alpha-h}{\beta-h} + \frac{\beta-h}{\gamma-h} + \frac{\gamma-h}{\alpha-h}$

$= \frac{-1-1}{2+\sqrt{3}i-1} + \frac{2+\sqrt{3}i-1}{2-\sqrt{3}i-1} + \frac{2-\sqrt{3}i-1}{-1-1}$

$= \frac{-2}{1+\sqrt{3}i} + \frac{1+\sqrt{3}i}{1-\sqrt{3}i} + \frac{1-\sqrt{3}i}{-2}$

$= \frac{-2(1-\sqrt{3}i)}{4} + \frac{(1+\sqrt{3}i)^2}{4} + \frac{(-2)(1-\sqrt{3}i)}{4}$
$= \frac{1}{4}[-2 + 2\sqrt{3}i + 1 - 3 + 2\sqrt{3}i - 2 + 2\sqrt{3}i]$
$= \frac{1}{4}[-6 + 6\sqrt{3}i]$
$= 3\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$

Question17

If x and y are two positive real numbers such that $x + iy = \frac{13\sqrt{-5+12i}}{(2-3i)(3+2i)}$, then $13y - 26x =$

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Options:

A. 28

B. 39

C. 42

D. 54

Answer: A

Solution:

Given, $\sqrt{-5+12i} = \sqrt{\frac{13-5}{2}} + i\sqrt{\frac{13+5}{2}}$

$= 2 + 3i$

$$\therefore x + iy = \frac{13(2+3i)}{(2-3i)(3+2i)}$$

$$= \frac{13(2+3i)}{6 - 9i + 4i - 6i^2} = \frac{13(2+3i)}{12 - 5i}$$

$$= \frac{13(2+3i)(12+5i)}{(12-5i)(12+5i)}$$

$$= \frac{13\left(24+36i+10i+15i^2\right)}{169}$$

$$= \frac{13(9+46i)}{169} = \frac{9}{13} + \frac{46i}{13}$$

$$\Rightarrow \quad x = \frac{9}{13} \text{ and } y = \frac{46}{13}$$

$13y - 26\dot{x} = 46 - 18 = 28$

Question18

If $z = x + iy$ and if the point P represents z in the argand plane, then the locus of z satisfying the equation $|z-1| + |z+i| = 2$ is

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Options:

A. $15x^2 - 2xy + 15y^2 - 16x + 16y - 48 = 0$

B. $3x^2 + 2xy + 3y^2 - 4x - 4y = 0$

C. $3x^2 - 2xy + 3y^2 - 4x + 4y = 0$

D. $15x^2 + 2xy + 15y^2 + 16x - 16y - 48 = 0$

Answer: C

Solution:

Given, $z = x + iy$

$|x + iy - 1| + |x + iy + i| = 2$

$\sqrt{(x-1)^2 + y^2} + \sqrt{x^2 + (y+1)^2} = 2$

$\sqrt{(x-1)^2 + y^2} = 2 - \sqrt{x^2 + (y+1)^2}$

$x^2 + y^2 - 2x + 1 = 4 + x^2 + y^2 + 2y + 1 - 4\sqrt{x^2 + (y+1)^2}$

$x + y + 2 = 2\sqrt{x^2 + y^2 + 2y + 1}$

$$x^2 + y^2 + 4 + 2xy + 4x + 4y$$
$$= 4x^2 + 4y^2 + 8y + 4$$

$3x^2 + 3y^2 - 2xy - 4x + 4y = 0$

Question19

One of the values of $(-64i)^{5/6}$ is

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Options:

A. $32i$

B. $16\sqrt{2}(1+i)$

C. $32(1+i)$

D. $16\sqrt{2}i$

Answer: B

Solution:

To find one of the values of $(-64i)^{5/6}$, let's proceed with the computation:

Let $Z = (-64i)^{5/6}$.

We can express Z as:

$Z = \left((-i)^5\right)^{1/6} \times (64)^{5/6}$

Calculate $(64)^{5/6}$**:**

$64^{5/6} = 32$

Handle the imaginary unit:

Since $(-i)^5 = -i$, we represent this in polar coordinates:

$-i = \cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)$

Find $(-i)^{1/6}$:

Using De Moivre's theorem, $(-i)^{1/6}$ can be expressed as:

$(-i)^{1/6} = \left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right]^{1/6}$

$= \cos\left(\frac{3\pi}{12}\right) + i\sin\left(\frac{3\pi}{12}\right)$

$= \frac{1}{\sqrt{2}}(1+i)$

Combine the results:

Therefore, Z becomes:

$Z = 32 \times \frac{1}{\sqrt{2}}(1+i) = 16\sqrt{2}(1+i)$

The calculation yields one of the values of $(-64i)^{5/6}$ as $16\sqrt{2}(1+i)$.

Question20

If $\frac{(2-i)x+(1+i)}{2+i} + \frac{(1-2i)y+(1-i)}{1+2i} = 1 - 2i$, then $2x + 4y =$

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Options:

A. 5

B. -2

C. 1

D. -1

Answer: A

Solution:

We have,

$$\frac{(2-i)x+(1+i)}{2+i} + \frac{(1-2i)y+(1-i)}{1+2i} = 1 - 2i$$

$$\Rightarrow \frac{(2x+1)+(1-x)i}{2+i}+\frac{(y+1)+(-2y-1)i}{1+2i}=1-2i$$

$$\Rightarrow \frac{[(2x+1)+(1-x)i](2-i)}{4-i^2}+\frac{[(y+1)+(-2y-1)i](1-2i)}{1-4i^2}=1-2i$$

$$\Rightarrow [2(2x+1)+(1-x)+(y+1)+2(-2y-1)]$$

$$+i[-2x-1+2(1-x)-2(y+1)+(-2y-1)]$$

$$=5(1-2i)$$

$$\Rightarrow (3x-3y+2)+i(-4x-4y-2)=5-10i$$

Question21

If $z=1-\sqrt{3}i$, then $z^3-3z^2+3z=$

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Options:

A. 0

B. $1+3\sqrt{3}i$

C. 1

D. $2+3\sqrt{3}i$

Answer: B

Solution:

We have,

$z=1-\sqrt{3}i=2e^{-i\frac{\pi}{3}}$

Now, z^3-3z^2+3z

$$=\left(2e^{-\frac{\pi}{3}i}\right)^3-3\left(2e^{-\frac{\pi}{3}i}\right)^2+3\left(2e^{-\frac{\pi}{3}i}\right)$$

$$=8e^{-\pi i}-12e^{-\frac{2\pi}{3}i}+6e^{-\frac{\pi}{3}i}$$

$$= 8(\cos(-\pi) + i\sin(-\pi))$$

$$-12\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$$

$$+6\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$= 8(-1 + i(0)) - 12\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$+6\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$= (-8 + 6 + 3) + i(0 + 6\sqrt{3} - 3\sqrt{3})$$

$$= 1 + 3\sqrt{3i}$$

Question22

The product of all the values of $(\sqrt{3} - i)^{\frac{2}{5}}$ is

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Options:

A. $2(\sqrt{3} - i)$

B. $2(\sqrt{3} + i)$

C. $2(1 - \sqrt{3}i)$

D. $2(1 + \sqrt{3}i)$

Answer: C

Solution:

Let, $(\sqrt{3} - i)^{2/5} = x$

$\Rightarrow \quad (\sqrt{3}-i)^2 = x^5$

$\Rightarrow \quad x^5 = (\sqrt{3})^2 + (i)^2 - 2\sqrt{3}i$

$\Rightarrow \quad x^5 = 3 + (-1) - 2\sqrt{3}i$

$\Rightarrow \quad x^5 = 2 - 2\sqrt{3}i = 2(1 - \sqrt{3}i)$

Clearly, the product of all the values of $(\sqrt{3}-i)^{2/5}$ is $2(1-\sqrt{3}i)$

Question23

The number of common roots among the 12 th and 30th roots of unity is

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Options:

A. 12

B. 9

C. 8

D. 6

Answer: D

Solution:

12 th roots of unity are $l, e^{i\frac{\pi}{6}}, e^{i\frac{2\pi}{6}}, e^{i\frac{3\pi}{6}}, e^{i\frac{4\pi}{6}}, \ldots, e^{i\frac{11\pi}{6}}$.

30th roots of unity are $1, e^{i\frac{\pi}{15}}, e^{i\frac{2\pi}{15}}, e^{i\frac{3\pi}{15}}, e^{i\frac{4\pi}{15}}, \ldots, e^{i\frac{29\pi}{15}}$.

Clearly, common roots are $1, e^{i\frac{\pi}{3}}, e^{i\frac{2\pi}{3}}, e^{i\pi}, e^{i\frac{4\pi}{3}}$ and $e^{i\frac{5\pi}{3}}$

Hence, the number of common roots is 6.

Question24

If $\sqrt{5}-i\sqrt{15} \div r(\cos\theta+i\sin\theta), -\pi<\theta<\pi$, then $r^2\left(\sec\theta+3\operatorname{cosec}^2\theta\right)=$

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Options:

A. 40

B. 60

C. 120

D. 180

Answer: C

Solution:

We have,

$$\sqrt{5}-i\sqrt{15}=r(\cos\theta+i\sin\theta)$$

$$\Rightarrow \quad r=\sqrt{(\sqrt{5})^2+(-\sqrt{15})^2}$$

$$=\sqrt{5+15}=2\sqrt{5}$$

$$\text{and } \theta=\tan^{-1}\left(\frac{\text{Imaginary part}}{\text{Real part}}\right)$$

$$=\tan^{-1}\left(\frac{-\sqrt{15}}{\sqrt{5}}\right)=\tan^{-1}(-\sqrt{3})$$

$$\Rightarrow \quad \theta=-\frac{\pi}{3}$$

$$\text{So, } r^2\left(\sec\theta+3\operatorname{cosec}^2\theta\right)$$

$$=(2\sqrt{5})^2\left[\sec\left(-\frac{\pi}{3}\right)+3\operatorname{cosec}^2\left(-\frac{\pi}{3}\right)\right]$$

$$=20\left[2+3\times\frac{4}{3}\right]=20\times 6=120$$

Question25

The point P denotes the complex number $z = x + iy$ in the argand plane. If $\frac{2z-i}{z-2}$ is a purely real number, then the equation of the locus of P is

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Options:

A. $2x^2 + 2y^2 - 4x - y = 0$

B. $x + 4y - 2 = 0$ and $(x, y) \neq (2, 0)$

C. $x - 4y - 2 = 0$ and $(x, y) \neq (2, 0)$

D. $x^2 + y^2 - 4x - 2y = 0$

Answer: B

Solution:

We have, $z = x + iy$

$$\therefore \frac{2z-i}{z-2} = \frac{2(x+iy)-i}{x+iy-2}$$

$$= \frac{2x+(2y-1)i}{x-2+iy}$$

$$= \frac{2x+(2y-1)i}{(x-2)+yi} \times \left[\frac{(x-2)-yi}{(x-2)-yi}\right]$$

$$2x(x-2)+y(2y-1)$$

$$= \frac{+[(x-2)(2y-1)-2xy]i}{(x-2)^2+y^2}$$

$$= \frac{2x^2-4x+2y^2-y}{(x-2)^2+y^2}$$

$$+ \frac{[(x-2)(2y-1)-2xy]i}{(x-2)^2+y^2}$$

$\because \frac{2z-i}{z-2}$ is purely real number

$$\Rightarrow \quad \frac{(x-2)(2y-1)-2xy}{(x-z)^2+y^2}=0$$

$$\Rightarrow \quad 2xy-x-4y-2xy+2=0$$

$$\Rightarrow \quad x+4y=2$$

Question26

x and y are two complex numbers such that $|x|=|y|=1$.

If $\arg(x)=2\alpha, \arg(y)=3\beta$ and $\alpha+\beta=\frac{\pi}{36}$, then $x^6y^4+\frac{1}{x^6y^4}=$

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Options:

A. 0

B. -1

C. 1

D. $\frac{1}{2}$

Answer: C

Solution:

Given x and y are two complex number with $|x|=|y|=1$ and $x=e^{i2\alpha}$

$$y=e^{i.3\beta}, \text{ where } \alpha+\beta=\frac{\pi}{36}$$

$$\Rightarrow \quad x^6=\left(e^{i2\alpha}\right)^6=e^{i.12\alpha}$$

$$y^4=\left(e^{i.3\beta}\right)^4=e^{i.12\beta}$$

So, $x^6 \cdot y^4=e^{i.12\alpha} \cdot e^{i,12\beta}=e^{i.12(\alpha+\beta)}$

Since, $\alpha+\beta=\frac{\pi}{36}$

$\Rightarrow \quad x^6 \cdot y^4=e^{i .12 \frac{\pi}{36}=e^{i \frac{\pi}{3}}}$

and $\frac{1}{x^6 \cdot y^4}=e^{-i \cdot \frac{\pi}{3}}$

$$\Rightarrow x^6 \cdot y^4+\frac{1}{x^6 \cdot y^4}=e^{i \cdot \frac{\pi}{3}}+e^{-i \cdot \frac{\pi}{3}}$$

$$=2 \cos \frac{\pi}{3}=1$$

Question27

One of the roots of the equation $x^{14}+x^9-x^5-1=0$ is

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Options:

A. $\frac{1+\sqrt{3} i}{2}$

B. $\frac{\sqrt{5}-1}{4}+i \frac{\sqrt{10-2 \sqrt{5}}}{4}$

C. $\frac{1-\sqrt{3} i}{2}$

D. $\frac{\sqrt{5}+1}{4}+i \frac{\sqrt{10-2 \sqrt{5}}}{4}$

Answer: D

Solution:

We have,

$x^{14}+x^9-x^5-1=0$

$\Rightarrow \quad\left(x^9-1\right)\left(x^5+1\right)=0$

$x^9=1$

$x=(1)^{1 / 9}$

$x=(\cos 0^{\circ}$

$+i \sin 0^{\circ})^{1/9}$

$=(\cos 2K\pi$

$+i \sin 2K\pi)^{1/9}$

$=\cos \frac{2K\pi}{9}+i \sin \frac{2K\pi}{9}$

using Demoivre's theorm here, $K=0,1,\ldots,8$

$x^{5}=-1$

$x=(\cos \pi+i \sin \pi)^{1/5}$

$=[\cos (2K\pi+\pi)$

$+i \sin (2K\pi+\pi)]^{1/5}$

$=[\cos (2K+1)\pi$

$+i \sin (2K+1)\pi]^{1/5}$

$=\cos \frac{(2K+1)\pi}{5}$

$+i \sin \frac{(2K+1)\pi}{5}$

where $K=0,1,\ldots,5$

So, one of root of the given equation is

So, one of root $\frac{2\pi}{9}+i \sin \frac{2\pi}{9}$ for $K=1$

$=\frac{\sqrt{5}+1}{4}+i \frac{\sqrt{10-2\sqrt{5}}}{4}$

Question28

$\operatorname{Arg}\left(\sin \frac{6\pi}{5} + i\left(1 + \cos \frac{6\pi}{5}\right)\right) =$

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Options:

A. $\frac{5\pi}{6}$

B. $\frac{6\pi}{5}$

C. $\frac{2\pi}{5}$

D. $\frac{9\pi}{10}$

Answer: D

Solution:

Given that

$\left(z = \sin\left(\frac{6\pi}{5}\right) + i\left(1 + \cos \frac{6\pi}{5}\right)\right)$

Thus, clearly $\left(\sin \frac{6\pi}{5}, 1 + \left(\cos \frac{6\pi}{5}\right)\right)$ lies in the 2 nd quadrant of complex plane. Hence, its argument is

$\arg(z) = \pi - \tan^{-1}\left|\left(\frac{y}{z}\right)\right|$
$(\because z = x + iy, \forall x < 0 \text{ and } y \geqslant 0)$

$$= \pi - \tan^{-1}\left|\frac{1 + \cos \frac{6\pi}{5}}{\sin \frac{6\pi}{5}}\right|$$
$$= \pi - \tan^{-1}\left|\frac{1 - \cos \frac{\pi}{5}}{\sin \frac{\pi}{5}}\right|$$

$$= \pi - \tan^{-1}\left|\frac{2\sin^2\frac{\pi}{10}}{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}\right|$$
$$= \pi - \tan^{-1}\left|\frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}}\right|$$
$$= \pi - \tan^{-1}\left(\tan\frac{\pi}{10}\right)$$
$$= \pi - \frac{\pi}{10}\left(\because -\frac{\pi}{2} \leq \tan^{-1} x \leq \frac{1}{2}\right)$$
$$= \frac{10\pi - \pi}{10} = \frac{9\pi}{10}$$
$$\text{Thus, } \arg(z) = \frac{9\pi}{10}$$

Question29

If $x + iy = \sqrt{\frac{3+i}{1+3i}}$, then $\left(x^2 + y^2\right)^2 =$

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Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Given,

$$x+iy=\sqrt{\frac{3+i}{1+3i}} \qquad \ldots(i)$$

$$\Rightarrow \overline{x+iy}=\sqrt{\overline{\frac{3+i}{1+3i}}}$$

$$\Rightarrow x-iy=\sqrt{\frac{3-i}{1-3i}} \qquad \ldots(ii)$$

On multiplying Eqs. (i) and (ii), we get

$$x^2+y^2=\sqrt{\frac{(3+i)(3-i)}{(1+3i)(1-3i)}}=\sqrt{\frac{9+1}{1+9}}=1$$

$$\Rightarrow\left(x^2+y^2\right)^2=(1)^2=1$$

Question30

If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then the locus of the point representing z in the Argand plane is

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Options:

A. a circle

B. a straight line

C. a parabola

D. an ellipse

Answer: B

Solution:

Now, Img $\left(\frac{2z+1}{iz+1}\right)=-2$

Now, let $z=x+iy$

$$\therefore \frac{2z+1}{iz+1}=\frac{2(x+iy)+1}{i(x+iy)+1}=\frac{(2x+1)+i2y}{(1-y)+ix}$$

$$=\frac{\{(2x+1)+i2y\}\{(1-y)-ix\}}{(1-y)^2+x^2}$$

$= \frac{(2x+1)(1-y)+i(2y-2y^2)-i(2x^2+x)+2xy}{(1-y)^2+x^2}$

From Eq. (i), we get

$$\frac{2y-2y^2-2x^2-x}{1+y^2-2y+x^2} = -2$$
$$\Rightarrow 2y-2y^2-2x^2-x = -2-2y^2+4y-2x^2$$
$$\Rightarrow \quad 2y-x+2-4y = 0$$
$$\Rightarrow \quad x+2y-2 = 0$$

which is a straight line.

Question31

If $i = \sqrt{-1}$, then $(1+i)^{10} + (1-i)^{10} =$

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Options:

A. -64

B. 64

C. 0

D. $64i$

Answer: C

Solution:

If $i = \sqrt{-1}$, then consider evaluating $(1+i)^{10} + (1-i)^{10}$.

Solution

We know that $i^2 = -1$.

First, let's compute $(1+i)^2$ and $(1-i)^2$:

Calculate $(1+i)^2$:

$(1+i)^2 = (1^2 + 2 \cdot 1 \cdot i + i^2) = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$

Calculate $(1-i)^2$:

$(1-i)^2 = (1^2 - 2 \cdot 1 \cdot i + i^2) = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$

Now, raise each result to the 5th power:

$\{(1+i)^2\}^5 = (2i)^5$

$(2i)^5 = 2^5 \cdot i^5 = 32 \cdot i^5$

Since $i^5 = i^4 \cdot i = 1 \cdot i = i$, it follows that:

$(2i)^5 = 32i$

$\{(1-i)^2\}^5 = (-2i)^5$

$(-2i)^5 = (-2)^5 \cdot i^5 = -32 \cdot i^5$

Similarly, $i^5 = i$, thus:

$(-2i)^5 = -32i$

Finally, sum these results:

$(1+i)^{10} + (1-i)^{10} = 32i - 32i = 0$

Thus, the final answer is 0.

Question32

If z_1 and z_2 are complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then the difference in the amplitude of z_1 and z_2 is

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Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. 0

Answer: D

Solution:

To solve the problem, we have the condition that the magnitude of the sum of two complex numbers, z_1 and z_2, equals the sum of their magnitudes:

$|z_1+z_2|=|z_1|+|z_2|$

We start by squaring both sides of this equation:

$$(|z_1+z_2|)^2=(|z_1|+|z_2|)^2$$
$$|z_1|^2+|z_2|^2+2|z_1||z_2|\cos(\theta_1-\theta_2)=|z_1|^2+|z_2|^2+2|z_1||z_2|$$

Upon simplifying, we have:

$$2|z_1||z_2|\cos(\theta_1-\theta_2)=2|z_1||z_2|$$

This implies:

$$\cos(\theta_1-\theta_2)=1$$

The condition $\cos(\theta_1-\theta_2)=1$ indicates that:

$$\theta_1-\theta_2=0$$

Therefore, the difference in amplitude (or argument) between z_1 and z_2 is:

$$\arg(z_1)-\arg(z_2)=0$$

Thus, the two complex numbers have the same direction or angle in the complex plane.

Question33

If $i=\sqrt{-1}$, then $1+i^2+i^4+i^6+\ldots\ldots+i^{2024}=$

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Options:

A. i

B. $-i$

C. 1

D. -1

Answer: C

Solution:

Given, $1+i^2+i^4+i^6+\ldots+i^{2024}$

$$=1+\left(i^2+i^4+i^6+\ldots+1012 \text{ terms }\right)$$
$$=1+(-1+1-1+1-\ldots+1-1+1)$$

i.e. 506 terms will be +1 and 506 terms will be $-1 = 1 + 0 = 1$

Question34

If $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real, then $\cos^3\theta + \sin^2\theta + \cos\theta + 1 =$

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Options:

A. 0

B. 1

C. 2

D. $\frac{3}{4}(2+\sqrt{2})$

Answer: C

Solution:

To solve the problem, let's first express the given complex number:

$z = \frac{1+i\cos\theta}{1-2i\cos\theta}$

To simplify this expression and ensure it is purely real, we'll multiply the numerator and denominator by the conjugate of the denominator:

$z = \frac{(1+i\cos\theta)(1+2i\cos\theta)}{(1-2i\cos\theta)(1+2i\cos\theta)}$

Recall that multiplying by the conjugate will result in a real denominator:

$z = \frac{1+2i\cos\theta+i\cos\theta-2\cos^2\theta}{1-(2i\cos\theta)^2}$

Simplifying the denominator, since $(2i\cos\theta)^2 = -4\cos^2\theta$, we get:

$1-(2i\cos\theta)^2 = 1+4\cos^2\theta$

Thus, the expression for z becomes:

$z = \frac{1+3i\cos\theta-2\cos^2\theta}{1+4\cos^2\theta}$

For z to be purely real, the imaginary part must be zero. The imaginary part is:

$\text{Im}(z) = \frac{3\cos\theta}{1+4\cos^2\theta} = 0$

This implies that $\cos\theta = 0$.

Substituting $\theta = \frac{\pi}{2}$, we note:

$\cos\theta = 0$

$\sin\theta = 1$

Now, substitute these into the expression $\cos^3\theta + \sin^2\theta + \cos\theta + 1$:

$\cos^3\theta + \sin^2\theta + \cos\theta + 1 = 0^3 + 1^2 + 0 + 1 = 2$

Therefore, the final result is 2.

Question35

If $\theta = \frac{\pi}{6}$, then the 10 th term of the series $1 + (\cos\theta + i\sin\theta)^1 + (\cos\theta + i\sin\theta)^2 + \ldots$.. is

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Options:

A. -1

B. $-i$

C. $\frac{1}{2} + \frac{\sqrt{3}i}{2}$

D. 1

Answer: B

Solution:

$T_{10} = (\cos\theta + i\sin\theta)^9$

Using De-Moivre theorem,

$$\begin{aligned} & \cos(9\theta) + i\sin(9\theta) \\ = & \cos\left(\frac{9\pi}{6}\right) + i\sin\left(\frac{9\pi}{6}\right) \\ = & \cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right) \\ = & 0 + i(-1) \\ = & -i \end{aligned}$$

Question36

If α and β are non-zero integers and $z = (\alpha + i\beta)(2 + 7i)$ is a purely imaginary number, then minimum value of $|z|^2$ is

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Options:

A. 0

B. 2809

C. 2808

D. 1

Answer: B

Solution:

To find the minimum value of $|z|^2$ where $z = (\alpha + i\beta)(2 + 7i)$ is purely imaginary, follow these steps:

First, express z in terms of real and imaginary components:

$z = (\alpha + i\beta)(2 + 7i) = 2\alpha + 7\alpha i + 2\beta i - 7\beta = (2\alpha - 7\beta) + (7\alpha + 2\beta)i$

Since z is purely imaginary, its real part must be zero:

$2\alpha - 7\beta = 0 \implies \alpha = \frac{7}{2}\beta$

Given that α and β are integers, β must be such that α is also an integer. Therefore, β should be chosen so that $\frac{7}{2}\beta$ is an integer. This implies β must be even, so assume $\beta = 2k$ for some integer k.

Substituting $\beta = 2k$ into $\alpha = \frac{7}{2}\beta$, we get $\alpha = 7k$.

Next, calculate $|z|^2$:

$|z|^2 = (7\alpha + 2\beta)^2 = (7(7k) + 2(2k))^2 = (49k + 4k)^2 = (53k)^2$

To minimize $|z|^2$, we minimize k. The smallest positive integer value for k is 1. Therefore, when $k = 1$, $|z|^2 = 53^2 = 2809$.

Thus, the minimum value of $|z|^2$ is:

$\boxed{2809}$
